MULTI-RESOLUTION SMOOTHING OF NURBS CURVES BASED ON NON-UNIFORM B-SPLINE WAVELETS

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ABSTRACT

As a rule, an energy method is widely adopted for b-spline curve smoothing, but this method has the disadvantages such as massive calculation, computation complexity and low efficiency. Compared with the energy method, multi-resolution smoothing approaches nicely overcome these obstacles. Presently, some researches have been conducted on multi-resolution smoothing, but these efforts mainly aimed at uniform or quasi-uniform b-spline curves. Uniform and quasi-uniform b-spline curves are just exceptional cases of NURBS curves. Multi-resolution smoothing for these types of curves mostly depend on uniform b-spline wavelets, so this smoothing method can't be directly applied to NURBS curves. In this paper, firstly, new non-uniform b-spline wavelets are created based on discrete b-spline basis functions in the light of the particularities of NURBS curves, the wavelet reconstruction and decomposition algorithms are provided. The wavelets, obviously, have greater flexibility and applicability than uniform b-spline wavelets because of their distance-independence of neighbor nodes in knot vectors. Then, this paper presents the multi-resolution smoothing method for NURBS curves based on the newly built wavelets. Lastly, an example is presented to confirm effectiveness of this multi-resolution smoothing method. Furthermore, the method can also be applied to NURBS surfaces if extended properly.

KEYWORDS

NURBS curve, Non-uniform B-spline wavelet, Multi-resolution Analysis, Smoothing.

1. INTRODUCTION

One of the major tasks in computer graphics and computer-aided design (CAD) is able to generate the particular characteristic of curves and surfaces called "visual pleasantness", "smoothness" or "fairness". This is a crucial problem in many areas of work but primarily in automobile, ship, and aircraft design, since "automobiles have the most refined body sculpture and the highest customer expectations and awareness". A curve will sometimes oscillate and cause unwanted inflexions which are difficult to deal with either during interpolating or approximating some data sets or starting from scratching in a CAD system. The problem of curve smoothness aims to eliminate oscillations and remove inflexions from the curve shape.

According to the researches of smoothing techniques that have been made, the methods of smoothing can be classified into two classes: global smoothing and local smoothing. The first method, proposed by Kjellander, is restricted to cubic parametric splines with uniform parameterization. This method is then extended by Poliakoff. The second one, a local smoothing technique has been built by Farin and Sapidis based on knot removal and knot insertion operation. Eck and Hadenfeld have developed a local energy minimization smoothing approach through the modification of a single control point iteratively.

In addition, Wang et al. proposes to smooth a planar curve by solving a non-linear system to satisfy monotone curvature variation conditions. Xu et al. present an algorithm smoothing planar cubic B-spline curves by target curvature plots. Ke and Li studies the NURBS curve smoothing based on nonuniform B-spline wavelets. However, a large amount of integral computation has to be made to get the Gram matrix for these nonuniform B-spline wavelets. Amati proposes a multi-level filtering approach for smoothing planar B-spline curves based on quasi-uniform B-spline wavelets in a uniform dyadic subdivision of the parametric domain. Compared with that of Farin and Sapidis based on monotone curvature, it is proved that this multiresolution smoothing method has more obvious advantage. However, Amati' method is restricted to endpoint interpolating quasi-uniform B-spline curves. This limitation prevents the method from being used in many applications, because uniform parameterization usually gives unacceptable curves.

NURBS curves have greater power, flexibility and precision than other parametric spline curves. Especially important is the ability of NURBS to represent conic curves precisely. Thus, with NURBS a modeling system can use a single internal representation for a wide range of curves and surfaces. This single characteristic of NURBS is key to developing a robust computer aided design system. As a result, NURBS has become the standard of much of the computer aided design and interactive graphics community.

In this paper, a new approach is introduced based on multiresolution analysis of cubic NURBS curves. The algorithm is based on a new kind of nonuniform semi-orthogonal B-spline wavelets. For a detailed presentation of wavelet transform and multiresolution analysis (MRA), Mallat and Chui are refered to. Its applications in computer graphics and CAD can be found in Finkelstein and Stollnitz et al.

Multiresolution analysis and wavelets provide useful and efficient tools for representing functions at multiple levels of detail. The wavelets developed in this paper depend upon generalization of the standard notion of multiresolution analysis. Rather than developing wavelets from scales and translates over regular domains, they are based on discrete B-splines and B-spline knot insertion algorithm.

The main advantage of the multiresolution smoothing algorithm is that it does not compute derivatives or integrals to find bad control points. The designers can manipulate the curves acting on each level of detail by a threshold function, or only considering a single or a set of control points on which to operate. The developed technique has the characteristic of local and global smoothing.

The paper is organized as follows. Section 2 gives the principle of nonuniform semi-orthogonal B-spline wavelets and the multiresolution mathematics representation of NURBS curves based on the new wavelets. The multiresolution decomposition and reconstruction algorithms for the proposed wavelets are presented in section 3. The multiresolution curve smoothing algorithm is developed based on the new wavelets in section 4. The conclusion is given in section 5.

2. NONUNIFORM SEMI-ORTHOGONAL B-SPLINE WAVELETS

Let $\boldsymbol{\tau}_0 \subset \boldsymbol{\tau}_1 \subset \cdots \subset \boldsymbol{\tau}_j \subset \cdots$ be nested sequences of nonuniform knot vectors in a finite interval [a, b], where $\boldsymbol{\tau}_j = [t_{0,j}, t_{1,j} \cdots t_{n_j+k,j}]$, $j = 0,1\cdots$ satisfy a monotone non-decreasing property, where a = $t_{0,j}$, and b = $t_{n_j+k,j}$. Suppose that $\{N_{i,k,j}(t)\}_{i=0}^{n_j}$ is normalized B-spline basis functions of order k determined by knot vector $\boldsymbol{\tau}_j$. Then, { $N_{0,k,j}, \cdots, N_{n_j,k,j}$ }, $j = 0,1\cdots$ constitute a nested basis function sequences of degree k-1; these nested function sequences span k order NURBS function space. They can be represented by symbols, that is, $V_0 \subset V_1 \subset \cdots \subset V_j \subset \cdots$.

Collect the basis functions of the V_j space, and make the vector $\boldsymbol{N}_j = [N_{0,k,j}, \cdots, N_{n_j,k,j}]$, then there exists such relationship:

$$\boldsymbol{N}_{j-1} = \boldsymbol{N}_j \boldsymbol{P}_j \tag{1}$$

where $\boldsymbol{P}_j$ is a $(n_j+1)\times(n_{j-1}+1)$ matrix. If f = $\boldsymbol{N}_{j-1}\boldsymbol{d}_{j-1}$ is a function in V_{j-1} space, according to $V_0 \subset V_1 \subset \cdots \subset V_j \subset \cdots$, it can be represented as f = $\boldsymbol{N}_j\boldsymbol{d}_j$ in the V_j space, where $\boldsymbol{d}_j = \boldsymbol{P}_j\boldsymbol{d}_{j-1}$. The space V_j can be expressed as the direct sum of the V_{j-1} and W_{j-1} space, i.e., $V_j = V_{j-1} \oplus W_{j-1}$ where W_{j-1} is the semi-orthogonal complement space of V_{j-1}. In other words, W_{j-1} consists of all the functions in the V_j space that are semi-orthogonal to those in the V_{j-1} space. Let $\{\psi_{i,k,j}\}_{i=0}^{m_{j-1}}$ be a set of basis of the W_{j-1} space, where $m_{j-1} = n_j - n_{j-1}$. Then $\{\psi_{i,k,j}\}_{i=0}^{m_{j-1}}$ can be made a set of non-uniform B-spline wavelets. Let $\boldsymbol{\psi}_j = [\psi_{1,k,j}, \cdots, \psi_{m_j,k,j}]$, there exists a $(n_j+1)\times m_{j-1}$ dimension matrix $\boldsymbol{Q}_j$ such that:

$$\boldsymbol{\psi}_{j-1} = \boldsymbol{N}_j \boldsymbol{Q}_j \tag{2}$$

The $\boldsymbol{P}_j$ and $\boldsymbol{Q}_j$ are called the reconstruction matrices of these non-uniform B-spline wavelets. Equation (1) and (2) can be combinatory, and be written as:

$$[\boldsymbol{N}_{j-1} \,|\, \boldsymbol{\psi}_{j-1}] = \boldsymbol{N}_j[\boldsymbol{P}_j \,|\, \boldsymbol{Q}_j] = \boldsymbol{N}_j \boldsymbol{M}_j \tag{3}$$

where $M_j = [P_j | Q_j]$. As M_j is a regular matrix of order n_{j+1}, then there is as following:

$$\boldsymbol{N}_j = [\boldsymbol{N}_{j-1} \,|\, \boldsymbol{\psi}_{j-1}]\boldsymbol{M}_j^{-1}$$

$$= [\boldsymbol{N}_{j-1} \,|\, \boldsymbol{\psi}_{j-1}]\left[\frac{\boldsymbol{A}_j}{\boldsymbol{B}_j}\right].$$

$$\left[\frac{\boldsymbol{A}_j}{\boldsymbol{B}_j}\right] = [\boldsymbol{P}_j \,|\, \boldsymbol{Q}_j]^{-1}, \tag{4}$$

where $\boldsymbol{A}_j$ is a $(n_{j-1}+1)\times(n_j+1)$ dimension matrix, and $\boldsymbol{B}_j$ a $m_{j-1}\times(n_j+1)$ dimension matrix. The matrices $\boldsymbol{A}_j$ and $\boldsymbol{B}_j$ are called the decomposition matrices of these non-uniform B-spline wavelets.

For any B-spline function f_j in the V_j space, i.e., $f_j=\boldsymbol{N}_j\boldsymbol{d}_j$, f_j can be decomposed into two parts: the global smoothing part f_{j-1} and the detail feature part g_{j-1}, that is, $f_j = f_{j-1}+g_{j-1} = \boldsymbol{N}_{j-1}\boldsymbol{d}_{j-1}+\boldsymbol{\psi}_{j-1}\boldsymbol{w}_{j-1}$, where

$$\boldsymbol{d}_{j-1}=\boldsymbol{A}_j\boldsymbol{d}_j,\ \ \boldsymbol{w}_{j-1}=\boldsymbol{B}_j\boldsymbol{d}_j \tag{5}$$

Thus, there exists

$$\boldsymbol{d}_j=[\boldsymbol{P}_j\,|\,\boldsymbol{Q}_j]\begin{bmatrix}\boldsymbol{d}_{j-1}\\ \boldsymbol{w}_{j-1}\end{bmatrix} \tag{6}$$

According to equation (5), there exists

$$\boldsymbol{d}_j=\boldsymbol{P}_j\boldsymbol{d}_{j-1}+\boldsymbol{Q}_j\boldsymbol{w}_{j-1} \tag{7}$$

The matrix $\boldsymbol{P}_j$ can be constructed through the Oslo B-spline knot insertion algorithm. The main challenge is how to construct the matrix $\boldsymbol{Q}_j$ corresponding to suitable semi-orthogonal non-uniform B-spline wavelets $\{\psi_{i,k,j}\}_{i=0}^{m_{j-1}}$.

The knot vector $\boldsymbol{\tau}=[t_0,\cdots,t_{n+k}]$ on a finite interval [a, b] defines a B-spline function space. If the knot vector $\boldsymbol{\tau}$ has been set, the basis functions $\{N_{i,k}\}_{i=0}^{n}$ would be determined uniquely. Any function $f=\boldsymbol{Nd}$ in this function space is decided by the vector $\boldsymbol{d}$ uniquely, where $\boldsymbol{N}=[N_{0,k},\cdots,N_{n,k}]$, $\boldsymbol{d}=[d_{0,}\cdots,d_n]^T$. Thus we can construct non-uniform semi-orthogonal B-spline wavelets by discrete inner product. The orthogonal relationship in the V_j space can be established by $<f,h>=\boldsymbol{d}^T\boldsymbol{s}$, where $f=\boldsymbol{Nd}$, and $h=\boldsymbol{Ns}$. Then, the construction of the matrix $\boldsymbol{Q}_j$ can avoid a large amount of integral computation relative to wavelet orthogonality defined by the inner product $<f,h>=\int_a^b f(t)h(t)dt$.

Theorem 1. If the matrix $\boldsymbol{Q}_j$ is a column full rank matrix, furthermore,

$$\boldsymbol{P}_j^T\boldsymbol{Q}_j=\boldsymbol{0} \tag{8}$$

where $\boldsymbol{0}$ is a $(n_{j-1}+1)\times m_{j-1}$ dimension zero-matrix, then the W_{j-1} space must be the semi-orthogonal complement space of the V_{j-1} space included in the V_j space, that is, $V_j=V_{j-1}\oplus W_{j-1}$, and $\boldsymbol{\psi}_{j-1}$ is the non-uniform semi-orthogonal B-spline wavelets in the W_{j-1} space.

Proof. Let $f_{j-1}=\boldsymbol{N}_{j-1}\boldsymbol{d}_{j-1}$, and $g_{j-1}=\boldsymbol{\psi}_{j-1}\boldsymbol{w}_{j-1}$, where $f_{j-1} \in V_{j-1}, \boldsymbol{d}_{j-1} = [d_{0,k,j-1}, \cdots, d_{n_{j-1},k,j-1}]^T$, $g_{j-1} \in W_{j-1}$, and $\boldsymbol{w}_{j-1} = [w_{0,k,j-1,}, \cdots, w_{m_{j-1},k,j-1}]^T$.

The discrete inner product between f_{j-1} and g_{j-1} is as following:

$$
\begin{aligned}
< f_{j-1}, \quad g_{j-1} >&= \boldsymbol{d}_{j-1}^T \boldsymbol{w}_{j-1} \\
&= \boldsymbol{d}_{j-1}^T \boldsymbol{P}_j^T \boldsymbol{Q}_j \boldsymbol{w}_{j-1} \\
&= 0
\end{aligned}
$$

Thus it can be seen the space W_{j-1} is discretely orthogonal to the V_{j-1} space, and **theorem 1** is true.

3. MULTIRESOLUTION DECOMPOSITION AND RECONSTRUCTION ALGORITHMS

3.1 The reconstruction algorithm

According to the contexts of section 2, the matrix $\boldsymbol{P}_j$ can be obtained by means of the Oslo algorithm. Then again, equation (8) implies that if the matrix $\boldsymbol{Q}_j$ is the solution of this homogeneous system of equations, the matrix $\boldsymbol{Q}_j$ must be column full rank matrix, and the W_{j-1} space exists as the semi-orthogonal complement space of the V_{j-1} space included in the V_j space. Thus the matrix $\boldsymbol{Q}_j$ can be completely determined by equation (8).

For any resolution level j, let $\boldsymbol{\tau} = \boldsymbol{\tau}_{j-1}$, $\bar{\boldsymbol{\tau}} = \boldsymbol{\tau}_j$, $n = n_{j-1}$, and $\bar{n} = n_j$. The reconstruction algorithm is as following.

Algorithm 1. Multiresolution reconstruction

Input: the order of B-spline curve: k, the resolution level: j-1, the coefficient vector corresponding to the j-1 resolution level: $\boldsymbol{d}_{j-1}$, and the wavelet coefficient vector: $\boldsymbol{w}_{j-1}$, and the knot vectors: $\boldsymbol{\tau}_{j-1}$ and $\boldsymbol{\tau}_j$.

Output: the reconstruction matrices: $\boldsymbol{P}_j$ and $\boldsymbol{Q}_j$; the coefficient vector of a higher resolution level than that of the input resolution level: $\boldsymbol{d}_j$.

The detailed procedures for its solution are as following:

(1) $\boldsymbol{\tau} = \boldsymbol{\tau}_{j-1}$, $\bar{\boldsymbol{\tau}} = \boldsymbol{\tau}_j$, $n = n_{j-1}$, and $\bar{n} = n_j$;

(2) To get $\boldsymbol{P}_j$ through the Oslo algorithm;

(3) To solve $\boldsymbol{Q}_j$ by Equation (8);

(4) To compute $\boldsymbol{d}_j$ by Equation (7).

3.2 The decomposition algorithm

After having the reconstruction matrices $\boldsymbol{P}_j$

and $\boldsymbol{Q}_j$, we can obtain $\boldsymbol{d}_{j-1}$ and $\boldsymbol{w}_{j-1}$ through Equation (5). However, the matrices $\boldsymbol{A}_j$ and $\boldsymbol{B}_j$ may be dense matrices noting that $\left[\frac{\boldsymbol{A}_j}{\boldsymbol{B}_j}\right]=[\boldsymbol{P}_j\,|\,\boldsymbol{Q}_j]^{-1}$. This would consume a great deal of computing cost, and reduce the efficiency of the algorithm. To avoid this disadvantage, an effective method is provided. First, the least square solution of the homogenous system equations $[\boldsymbol{P}_j\,|\,\boldsymbol{Q}_j]\boldsymbol{X}=\boldsymbol{d}_j$ can be obtained after calculating and analyzing, then the decomposition coefficient vector $\boldsymbol{d}_{j-1}$ and $\boldsymbol{w}_{j-1}$ can be gotten through the coefficient vector $\boldsymbol{d}_j$ in terms of their dimension correlation. The following is the material algorithm.

Algorithm 2. Multiresolution decomposition

Input: the order of B-spline curve: k, the resolution level: j, the coefficient vector corresponding to the resolution level j: $\boldsymbol{d}_j$, and the reconstruction matrices $\boldsymbol{P}_j$ and $\boldsymbol{Q}_j$.

Output: the coefficient vector $\boldsymbol{d}_{j-1}$ of lower resolution level than that of the input resolution level and the coefficient vector $\boldsymbol{w}_{j-1}$ corresponding to detail feature. The computing process is as following:

First, solve the following homogeneous linear system equations according to equation (6), i.e.,

$$[\boldsymbol{P}_j\,|\,\boldsymbol{Q}_j]\boldsymbol{X}=\boldsymbol{d}_j,$$

then the vector $\boldsymbol{X}$ can be obtained. In the terms of dimension correlation of the vectors $\boldsymbol{d}_{j-1}$ and $\boldsymbol{w}_{j-1}$, the vector $\boldsymbol{X}$ can be written as:

$$\boldsymbol{X}=\left[\frac{\boldsymbol{d}_{j-1}}{\boldsymbol{w}_{j-1}}\right],$$

where $\boldsymbol{d}_{j-1}$ is a column vector solution of dimension $(n_{j-1}+1)\times 1$, and $\boldsymbol{w}_{j-1}$ is that of dimension $m_{j-1}\times 1$.

4 MULTI-RESOLUTION SMOOTHING

NURBS curves play an important role in industrial product design and geometric modeling, such as using NURBS curves to start the modeling of a more complex shape (3D surfaces) in car body design. This section is dedicated to a new multi-resolution smoothing method based on non-uniform semi-orthogonal B-spline wavelets created newly in this paper.

The multi-resolution representation of a NURBS B-spline curve allows decomposing a curve into two parts including its global approximation and local details. The local details indicate the geometric feature or high frequencies of this curve that are involved in curve smoothing. In this methodology, smoothing for NURBS curves means that how to control and adjust high frequencies of the curves so as to get expected smoothing curves.

Considering the NURBS curve $f_j(t)$ with arbitrary knot vector $\boldsymbol{\tau}_j = [t_{0,j}, t_{1,j} \cdots t_{n_j+k,j}]$ in a limit domain [a, b], it follows that:

$$f_j(t) = f_{j-1}(t) + g_{j-1}(t)$$

$$g_{j-1}(t) = f_j(t) - f_{j-1}(t) = \boldsymbol{\psi}_{j-1}\boldsymbol{w}_{j-1}$$

$$= \boldsymbol{N}_j \boldsymbol{Q}_j \boldsymbol{w}_{j\text{-}1} \qquad (9)$$

It is obvious that g_{j-1} is a linear combination of wavelet coefficients $w_{i,k,j-1}$. By computing this curve's module $|g_{j-1}(t)|$ in its defined domain, this curve's segments that need to be smoothed can be confirmed completely.

According to the B-spline wavelet multi-resolution characteristics, the global approximation at the lowest level is the smoothest approximation of the original curve, and it represents the basic structure of the original curve. This implies that wavelet coefficients are only related to the features or high frequencies of the original curve. So the curve smoothing can be realized through controlling wavelet coefficients related to curve features, and the curve's basic structure is, meanwhile, kept unaltered. The selection of bad control points is determined by $|g_{j-1}(t)|$ at the level j-1 which can be used to verify which knot interval has maximum module, likewise applied for the levels j-2, j-3, ⋯. As shown in the Equation (9), if max $\{|g_{j-1}(t)|\} \in [t_i, t_{i+1}]$ implies that the ith control point at the level j-1 requires the largest offset to reach the best fitting to the original curve and the curve is not smooth in this interval. The offset of wavelet coefficients can be applied to both the single ith control point and its p neighbors.

The smoothing process is described as follows:

Build a threshold function $\mathrm{T}(\boldsymbol{w} \in R^n, \lambda \in R, m \in I, p \in I) \rightarrow R^n$ which receives a vector of wavelet coefficients, a threshold coefficient, an ordinal of the bad point and its neighboring control point number involved in smoothing as inputs, such as $\boldsymbol{w}$ = $\boldsymbol{w}_{j-1} = [w_{0,k,j-1,}, \cdots, w_{m_{j-1},k,j-1}]^T$, and the threshold coefficient $\lambda \in [0,1]$, the mth control point and its p neighboring control points, such that:

(1) The threshold type is global:

$$\mathrm{T}(\boldsymbol{w}_{j-1}, \lambda, m, p) = \boldsymbol{w}_{j-1} - \lambda \boldsymbol{w}_{j-1} \qquad (10)$$

(2) The threshold type is local:

$$\mathrm{T}(\boldsymbol{w}_{j-1}, \lambda, m, p) = \boldsymbol{w}_{j-1} - \lambda \boldsymbol{w}'_{j\text{-}1} \qquad (11)$$

$$\boldsymbol{w}'_{j\text{-}1} = [0, \cdots, w_{m-p,k,j-1}, \cdots, w_{m+p,k,j-1}, 0, \cdots]^T$$

Set the value of λ in the reconstruction process and this value can be different each other at different reconstruction levels. The parameters m and p is determined by the module of $g_{j-1}(t)$.

Apply the threshold function $\mathrm{T}(\boldsymbol{w}, \lambda, m, p)$ to those curves' segments that $|g_j(t)|$ ($j = j-1, j-2, \cdots$) has the largest magnitude to filter out the undesired features of the curves, or only at those wavelet coefficients related to bad control points. The detailed algorithm is as following:

Algorithm 3. Multiresolution smoothing

```
call Algorithm 2 in order to get
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$(\boldsymbol{d}_{j-1}, \boldsymbol{w}_{j-1})$;

while (!($|g_{j-1}| < \varepsilon$))

if Threshold-Type == global **then**

call equation (10) to get new $\boldsymbol{w}_{j-1}$;

else

(m, p) = *FindBadKnot*($g_{j-1}(t)$);

call equation (11) to get new $\boldsymbol{w}_{j-1}$;

end if

do while

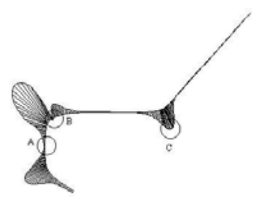
call **Algorithm 1** to get f_j;

draw f_j.

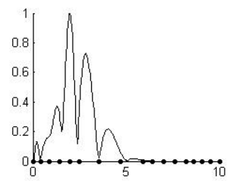
In **Algorithm 3**, the function *FindBadKnot*($g_{j-1}(t)$) is used to achieve the parameters m and p , that is , this function is mainly to compute the max $\{|g_{j-1}(t)|\} \in [t_i, t_{i+1}]$.

According to Algorithm 3, some results obtained are discussed through this multiresolution smoothing technique based on the non-uniform B-spline wavelets newly created as showed in Figure 1. The curve we considered is a cubic NURBS curve with 19 control points, i.e. $f_4(t)$ in Figure a. This curve's curvature plot is displayed in Figure b. From Figure b, abrupt curvatures exist in the place A and B. The module of this curve's detail feature $g_3(t)$ reveals problem in segments $[t_7, t_8]$, and feature coefficients related to control point $\boldsymbol{P}_7$ and its neighbors $\boldsymbol{P}_6$ and $\boldsymbol{P}_8$ are all needed to make threshold control as shown in Figure c. The smoothing result is as shown in Figure d by threshold calculation. Obviously, the smoothing property of the NURBS curve has been improved significantly. The module of this curve's detail feature $g_2(t)$ indicates that the feature coefficients related to control point $\boldsymbol{P}_4$ and its neighbors $\boldsymbol{P}_3$ and $\boldsymbol{P}_5$ are all needed to make threshold control as shown in Figure e. Figure f is the result after making threshold calculation and adjusting, which shows this curve's curvature is continuous, and no inflection points. After twice smoothing made, this NURBS curve can fully meet the demand for smoothness.

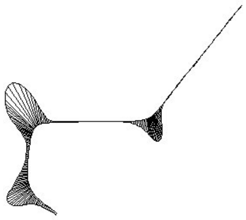
(a) A cubic NURBS curve $f_4(t)$ with 19 control points.



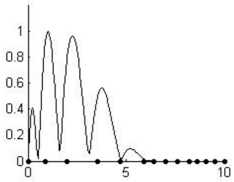
(b) The curve's curvature plot.



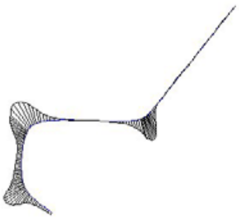
(c) The module of the detail feature $g_3(t)$



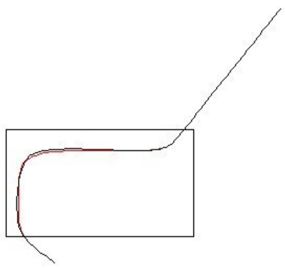
(d) Curvature plot after first multiresolution smoothing



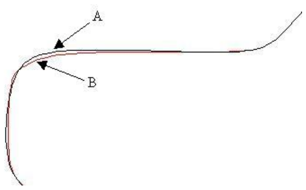
(e) The module of the detail feature $g_2(t)$



(f) Curvature plot after second multiresolution smoothing



(g) Comparative analysis between the original curve and the multi-resolution smoothing curve newly created



(h) The local enlarged drawing in Fig. g

A－the original curve

B－the smoothed curve

Figure 1. Multi-resolution smoothing for a cubic NURBS curve

5 CONCLUSIONS

This paper presents a new multi-resolution smoothing method for NURBS curves based on non-uniform semi-orthogonal B-spline wavelets newly created. This kind of wavelets is not limited by knot distribution in a finite interval, and is suitable for multi-resolution analysis of any uniform or non-uniform B-spline curves. This wavelets avoid the limitation of uniform B-spline wavelets for extensive applications of multi-resolution curve smoothing, and reduces computing burdensomeness of other non-uniform B-spline wavelets in multi-resolution smoothing.

In this approach, the selection of bad points differs from others that depend on the computation of energy integrals, non-linear optimization or curvature derivatives, rather depends on the maximum of wavelet function module. The new multi-resolution technique accomplishes a simple and efficient curve smoothing. Global or local smoothing of B-spline curves can be performed through a threshold function. At the end of this paper, the example presented confirms the effectiveness of this multi-resolution smoothing method. Future work will concern the generalization of this multiresolution smoothing technique extended to NURBS surfaces.

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